Accurate Materials Predictions with DFT & Machine Learning

Noa Marom
Materials Science & Engineering

Carnegie Mellon University
Machine Learning in Materials Simulations

Machine learning: A statistical model is built based on available “training” data to predict the results of future experiments

Applications in computational materials science:
• Machine learned inter-atomic potentials
• Machine learned DFT functionals
• Clustering
• Identifying correlations in data
• Feature selection
• Property prediction
• Optimization (e.g., Bayesian optimization)

Ingredients:
• Training data
• Representation
• Model type
• Model hyperparameters
• Validation

ML models can only interpolate, not extrapolate

It may be challenging to learn from “small data”. Incorporating physical knowledge into models can help

The application of ML models in materials simulations is usually not “black box” and some customization is required
A Machine Learned Model for Molecular Crystal Volume Estimation
Molecular Crystals

Used for *e.g.*, pharmaceuticals, organic electronics

Weak dispersion (van der Waals) interactions produce potential energy landscapes with many local minima close in energy

Molecular crystals often exhibit **polymorphism**, the ability of the same molecule to crystallize in several structures

Polymorphs may have different physical/chemical properties!

The challenge: given a 2D stick diagram of a molecule, predict all of its possible polymorphs

Requires searching a high-dimensional space with a high accuracy
The molecular solid form volume is the effective volume occupied by a molecule in a crystal:

\[ V_M = \frac{V_{cell}}{Z} \]

Crystal structure prediction workflows often begin by estimating the solid form volume to define the search space.

Workflow of the Genarris random structure generator for molecular crystals:

We developed a machine learned model to predict \( V_M \), given the single molecule structure.

ML Model for Volume Estimation: Training Data

The performance of ML models depends on the quality of the training data. The data should be consistent and reliable.

A set of polymorphic crystal structures characterized in ambient temperature and pressure conditions was extracted from CSD 2019. Porous structures were removed. Problems, such as discrepancies in Z values and chemical formula were corrected.

The final training set contained 2,472 unique pairs of polymorphs.

The standard deviation of the percent density difference between polymorphs may be considered as a lower bound for the error of a ML model.

The ML model is based on a combination of geometric and chemical descriptors that capture the salient features of molecular crystals.

**Geometric descriptor:** volume enclosed by the packing accessible surface.

**Chemical descriptor:** molecular topological fragments.

---

The predicted solid form volume is given by:

\[ V_M = \beta_0 V_0 + \sum_{i=1}^{n} \beta_i f_i \]

The coefficients are found by minimizing the ridge regression loss function:

\[ L(\beta) = \sum_{j=1}^{N} (V_{CSD,j} - V_{M,j})^2 + \lambda \sum_{i=0}^{n} \beta_i^2 \]

The ML model has three hyper-parameters:
- Number of molecular topological fragments
- Probe radius for packing-accessible surface construction, \( \alpha \)
- Ridge regression regularization parameter, \( \lambda \)

- The parameters were optimized by a 3D grid search over 54,810 combinations
- 10-fold cross validation was performed for each set of parameters
- Optimal values found: 2,231 fragments; \( \alpha = 3 \, \text{Å} \); \( \lambda = 10 \)

The model performs well for the training set and three sets of unseen data with errors below the presumed lower bound.

ML Model for Volume Estimation: Results

The volume enclosed by the packing-accessible surface captures the effect of sterically hindered regions and voids.

A model based only on the volume enclosed by the packing-accessible surface, without chemical information, has a broader error distribution.

Outliers include materials with strong attractive interactions, such as H-bonds, repulsive groups, such as halogens, N lone-pairs, and alkyl side chains.

A model based only on topological fragments, without volume information has a broader error distribution.

Outliers have sterically hindered regions, groups that do not participate in intermolecular interactions, or conformational polymorphs.

Including both geometric and chemical information is essential to the performance of the ML model.

I. Bier and N. Marom
Machine Learning the Hubbard U Parameter in DFT+U
A hybrid interface between two dissimilar materials may exhibit unique physical properties that do not exist in either bulk material.

Spin injection at an interface between a ferromagnet and a semiconductor enables the implementation of a spin valve.

T. A. Peterson et al., Phys. Rev. B 94, 235309 (2016);

A superconductor/semiconductor interface may enable the realization of networks of qubits based on Majorana zero modes.

J. Shabani et al., Phys. Rev. B 93, 155402 (2016);

Our goal is to develop computational tools for predicting the structure and properties of hybrid interfaces.
Many DFT codes are based on plane-wave basis sets and therefore impose 3D periodic boundary conditions.

The interface must be commensurate in the x-y plane, which may require large supercells.

Often, a large number of layers of each material is needed to avoid quantum confinement effects.

For a surface, vacuum space must be added along z to avoid spurious interactions between periodic replicas.

Hydrogen passivation of dangling bonds at the surface may be required to eliminate spurious states.

DFT simulations of interfaces are technically involved and computationally expensive!
Band Structure of InAs

The Perdew-Burke-Ernzerhof (PBE) generalized gradient approximation:


• Includes a dependence on the density and its gradient (semi-local functional)
• Computationally efficient
• Suffers from the self-interaction error
PBE produces no band gap for InAs

The Heyd-Scuzeria-Ernzerhof range-separated hybrid functional (HSE)


• A fraction of exact (Fock) exchange is mixed with the PBE exchange and correlation
• The Coulomb potential is split into short-range (SR) and long-range (LR) parts
• Has 25% exact exchange in the SR and reduces to PBE in the LR
HSE mitigates SIE and produces a gap for InAs but at a high computational cost
DFT+U(BO)

DFT+U

A Hubbard-like term, $U_{\text{eff}} = U - J$, is added to the DFT energy, where $U$ is the on-site Coulomb repulsion interaction and $J$ is the exchange interaction:

$$E_{\text{tot}} = E_{\text{DFT}} + \frac{U - J}{2} \sum_{\sigma} n_{m,\sigma} - n_{m,\sigma}^2$$


Offers a balance of accuracy and efficiency  $U_{\text{eff}}$ is a system dependent parameter

We machine learn $U_{\text{eff}}$ by Bayesian optimization (BO)

The objective function is formulated to reproduce the HSE band gap and band structure as closely as possible:

$$f(\bar{U}) = -\alpha_1 (E_{g}^{\text{HSE}} - E_{g}^{\text{PBE+U}})^2 - \alpha_2 (\Delta Band)^2$$

$$\Delta Band = \left( \frac{1}{N_E} \sum_{i=1}^{N_k} \sum_{j=1}^{N_b} (\varepsilon_{HSE}^i[k_i] - \varepsilon_{PBE+U}^i[k_i])^2 \right)^{1/2}$$

DFT+U(BO)

2D BO is performed to find the optimal U values for In-\( p \) and As-\( p \)

Negative values of U are allowed

PBE+U(BO) produces a comparable band structure to HSE at a fraction of the computational cost

M. Yu, S. Yang, C. Wu, and N. Marom, npj Computational Materials 6, 180 (2020)
Electronic Structure of InAs and InSb Surfaces

The parameters obtained for bulk InAs are transferrable to a surface slab with 11 layers (largest we could calculate with HSE)

M. Yu, S. Yang, C. Wu, and N. Marom, npj Computational Materials 6, 180 (2020)

40-50 atomic layers are required to converge the electronic structure of InAs and InSb surfaces to the bulk limit

S. Yang et al., arXiv 2012.14935 (2020)
A slab with 20 layers is used to simulate the $\beta_2(2\times4)$ reconstruction of InAs(001).

Unfolding in the xy plane onto a 1x1xZ slab produces a dense band structure.

Z-unfolding onto a bulk unit cell oriented in (001) yields more bulk-like band structure. Bands corresponding to $k_z=|\Gamma X|$ are present.

Z Unfolding

Bulk unfolding onto the primitive cell eliminates $k_z=|\Gamma X|$ bands. The band structure is in agreement with ARPES.

XY Unfolding

Bulk Unfolding

arXiv 2012.14935
InAs(001) Surface Reconstructions

LEED shows superposition of 2x4 and 4x2 reconstructions

Different reconstructions exhibit different signatures of surface states but have similar band bending

DFT supports the coexistence of 2x4 and 4x2 domains

Surface sensitive ARPES would be needed to detect surface states

S. Yang et al., arXiv 2012.14935 (2020)
Effect of Oxidation on InAs(111) vs InSb(110)

PBE+U(BO) is in agreement with ARPES experiments

For InAs(111) oxidation leads to band bending and the appearance of an electron pocket

For InSb(110) oxidation does not cause band bending and no electron pocket appears

This is due to stronger charge transfer from surface As to O than from Sb to O

Acknowledgements

Fall 2019

Maituo Yu
Chunzhi Wu
Manny Bier
Shuyang Yang

Download PyMoVE: https://github.com/manny405/PyMoVE
Download GAitor, Genarris, and Ogre: www.noamarom.com